2018

Probability of Default implied Rating

White Paper

The Credit Research Initiative (CRI)
National University of Singapore

First version: March 2, 2017, this version: November 29, 2018

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ABSTRACT

Introduced by the Credit Research Initiative (CRI) in 2011, the Probability of Default Implied Rating (PDiR) complements the CRI 1-year Probability of Default (PD) by providing a convenient and intuitive overview on the credit quality of a firm through the mapping of the CRI PD into letter grades used by major rating agencies. Details of the methodological foundations and numerical realization are presented in Addendum 3 to the RMI-CRI Technical Report (Version 2017). This white paper seeks to provide a more intuitive explanation of the PDiR, using a methodological walkthrough and an illustration.

CONTENT

* Please cite this document in the following way: "The Credit Research Initiative of the National University of Singapore (2018), Probability of Default implied Rating (PDiR) White Paper", Accessible via the following link: https://www.rmicri.org/en/white_paper/.

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I. OVERVIEW

Credit ratings in the form of alphabetical letter-grades are intended to give users a convenient and intuitive overview of an obligor's creditworthiness. The CRI Probability of Default Implied Rating (PDiR) was introduced in 2011 to complement the high-granularity CRI Probability of Default (CRI PD) by assigning a letter-grade to each firm according to a systematic mapping of 1-year PD based on historically observed default rates from Standard & Poor's (S&P) credit ratings. The methodology is revised and implemented on December 15, 2017 to provide a better match to the average default rate of S&P rating portfolio and the new PDiR are also provided with a plus or minus modifier when appropriate. The PDiR boundaries will be updated periodically to incorporate significant changes in the corporate credit markets.

The CRI 1-year PD is a numerical measure of the firm's default risk over the coming year. The PDiR segregates firms into alphabetical categories by mapping their PD to the equivalent letter grades by benchmarking to the default experience of the S&P rating system as reflected in the historical observed default rates of its global corporate rating pool. In short, the PDiR provides a letter grade to a firm's 1-year forward looking credit quality, using the S&P classification. Table 1 presents the CRI PDiR mapping table as of May 7, 2018. For example, a firm having its CRI 1-year PD in the range between 0 and 0.85 bps can be understood as a firm with similar creditworthiness as a representative S&P AAA rated firm.

Table 1. Conversion table of 1-year PD to PDiR mapping table

As of May 2018

<table>
<tr><th colspan="2">Rating Category</th><th>Observed S&P Average Default Rate (bps)</th><th>Smoothed S&P Average Default Rate (bps)</th><th>CRI PD lower bound (in bps)</th><th>CRI PD upper bound (in bps)</th></tr>
<tr><td colspan="2">AAA</td><td>0.00</td><td>0.29</td><td>0.00</td><td>0.85</td></tr>
<tr><td colspan="2">AA+</td><td>0.00</td><td>1.10</td><td>0.85</td><td>1.32</td></tr>
<tr><td colspan="2">AA</td><td>2.20</td><td>1.71</td><td>1.32</td><td>2.51</td></tr>
<tr><td colspan="2">AA-</td><td>3.85</td><td>2.65</td><td>2.51</td><td>2.80</td></tr>
<tr><td colspan="2">A+</td><td>6.00</td><td>4.12</td><td>2.80</td><td>5.58</td></tr>
<tr><td colspan="2">A</td><td>7.90</td><td>6.39</td><td>5.58</td><td>6.89</td></tr>
<tr><td colspan="2">A-</td><td>7.20</td><td>9.92</td><td>6.89</td><td>12.19</td></tr>
<tr><td colspan="2">BBB+</td><td>11.60</td><td>15.40</td><td>12.19</td><td>19.24</td></tr>
<tr><td colspan="2">BBB</td><td>18.50</td><td>23.89</td><td>19.24</td><td>32.36</td></tr>
<tr><td colspan="2">BBB-</td><td>31.10</td><td>37.04</td><td>32.36</td><td>38.85</td></tr>
<tr><td colspan="2">BB+</td><td>34.55</td><td>57.39</td><td>38.85</td><td>71.61</td></tr>
<tr><td colspan="2">BB</td><td>52.55</td><td>88.82</td><td>71.61</td><td>103.00</td></tr>
<tr><td colspan="2">BB-</td><td>105.80</td><td>137.23</td><td>103.00</td><td>173.96</td></tr>
<tr><td colspan="2">B+</td><td>199.65</td><td>211.46</td><td>173.96</td><td>258.02</td></tr>
<tr><td colspan="2">B</td><td>475.90</td><td>324.52</td><td>258.02</td><td>403.54</td></tr>
<tr><td colspan="2">B-</td><td>846.05</td><td>494.98</td><td>403.54</td><td>651.30</td></tr>
<tr><td rowspan="5">CCC/C</td><td>CCC+</td><td rowspan="5">2,719.00</td><td>748.06</td><td>651.30</td><td>778.97</td></tr>
<tr><td>CCC</td><td>1,115.34</td><td>778.97</td><td>1,348.98</td></tr>
<tr><td>CCC-</td><td>1,631.16</td><td>1,348.98</td><td>2,036.17</td></tr>
<tr><td>CC</td><td>2,323.16</td><td>2,036.17</td><td>2,458.54</td></tr>
<tr><td>C</td><td>4,217.99</td><td>2,458.54</td><td>10,000.00</td></tr>
</table>

For those who are interested in PDiR mapping table benchmarking to historically observed default rates from Moody's credit ratings, please kindly refer to table 2 in Appendix.

Coverage

The CRI provides PDiR credit ratings for every public firms under its PD coverage. Correspondingly, this means PDiR covers the same sample as the CRI PD, for over 67,000 public-listed firms in 128 economies around the world. PDiR are also updated daily for over 34,000 actively listed firms within this database.

II. METHODOLOGY

Mapping the CRI 1-year PD to the respective PDiR requires defining the upper bound for all rating categories. The boundary values of the PDiR categories are obtained by first smoothing the average realized 1-year default rates for the specific rating category (ADR) of the S&P global corporate rating pool followed by searching for the boundary values that best match the empirical distribution of the CRI 1-year PD and the category-specific smoothed observed ADR. These boundary values will be recalibrated periodically to reflect changes in the credit markets. A detailed description of the procedure and methodology can be found in the Addendum 3 to the RMI-CRI Technical Report (2017).

S&P publishes the default rates for rating categories and years in their Annual Global Corporate Default Study and Rating Transitions[1] annually. CRI computes the ADR over the most recent 20 years. The version on the date of this writing is based on data published in the S&P report for 2016.

Due to the lack of observed defaults for S&P AAA and AA+ rated firms and data on the individual subcategories of CCC/C (i.e., CCC+, CCC, CCC-, CC, and C), the boundary values for these categories could not be determined without first extrapolating/interpolating ADR for these categories. Our approach is to conduct a linear regression on logit-transformed ADR for the categories with meaningful values, and through which predict ADR for others. Specifically,

$$\text{Logit}(ADR_p) = \log\left(\frac{ADR_p}{1 - ADR_p}\right)$$

[1] Table 9, One-Year Global Corporate Default Rates By Rating Modifier, S&P 2016 Annual Corporate Default Study and Rating Transitions

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where p represents a certain rating category and ADR_p is the average one-year realized default rate of the category p.

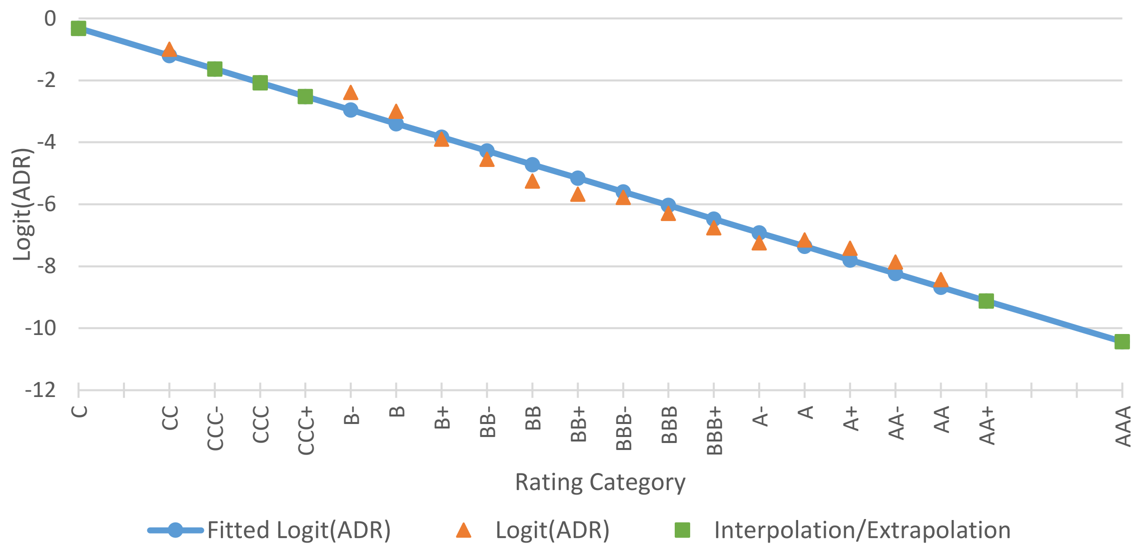

Fig 1. Logit (ADR) and rating categories

The boundaries of the extreme credit rating categories are obtained via either extrapolation or intrapolation.

Figure 1 presents the linear regression line used to smooth the relationship between the logit transformed ADR and the rating categories where we have assumed the S&P reported ADR for the combined category of CCC/C can be used to represent the CC category. Note that we are only able to obtain the realized default rate for this combined category. Moreover, the spacing between AAA and AA+ is assigned three ticks whereas that between CC and C is two ticks. These two assumptions do not affect the linear regression estimation but affect the interpolated/extrapolated values. The design is motivated by the fact that without extra spacing, the PDiR would have led to a far greater number of AAA or C firms as compared to the S&P rating practice.

The PDiR seeks to match the smoothed ADR for a specific rating category suggested by the regression line with the average probability of default (APD) using the empirical distribution of the 1-year PD in the CRI universe of exchange-listed firms. To accomplish this, we need to find PD cut-off values for each rating category.

The empirical distribution is constructed with snapshots of the CRI PD for the active firms at the year end for each of the previous two decades. The final empirical distribution is the average of the 20 distributions. The APD for a rating category can then be computed by averaging the PD of all the firms with their PD falling between the upper and lower boundary values for that rating category and the suitable boundary values will be chosen. Figure 2 below presents the empirical distribution of the CRI PD, the upper/lower bounds and the smoothed ADR for some rating categories.

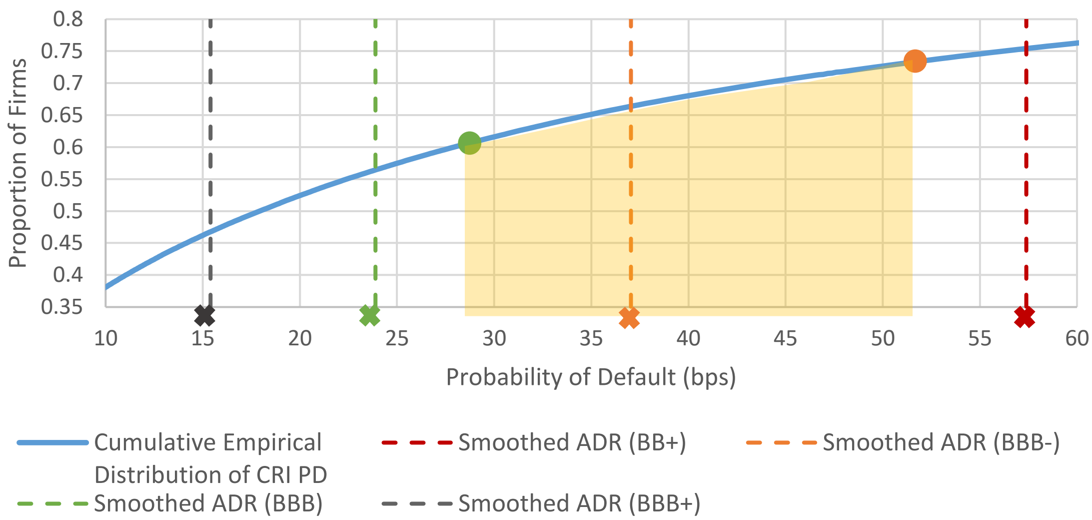

Fig 2. Empirical distribution of CRI 1-year PD, boundary values, and smoothed ADR

Because PD only take values between 0 and 10,000 bps, the lower bound for the best performing category and the upper bound for the worst performing category are naturally set. The remaining boundary values are then selected between two adjacent smoothed ADR by minimizing the sum of the squared relative differences between the APD and ADR over all rating categories, where the ADR is used as the base for computing the relative difference. For illustration, the yellow shaded area in Figure 2 defines the range of PD for the BBB- category, and the points marked by dots are the two boundary values. The goal is to find these two boundary values together with others so that the average PD confined to this range is as close to the smoothed ADR for the BBB- category as well as those for other rating categories.

Since there are 21 rating categories, 20 unknown boundary values are to be solved together. Mathematically, these 20 boundary values influencing APD are chosen to minimize the following objective function:

$$\sum_{p \in \{AAA, AA+, \ldots, C\}} \left(\frac{APD_p - \widehat{ADR}_p}{\widehat{ADR}_p} \right)^2$$

where $\widehat{ADR}_p$ represents the smoothed ADR for rating category p. We obtain the optimal solution by a sequential Monte Carlo algorithm.

Handling Spurious Credit Rating Migration

Large PD variations that cause a firm to move into a different risk category are informative as they give an update on the firms' financial health. However, slight PD variations may also trigger spurious credit rating migrations, i.e. a shift in credit rating when the value of the firm's PD is on or near the boundary value of two credit rating categories. Naturally, one would like to avoid credit rating shifts due to a minute transitory PD variation being hardcoded into a rating change. In order to reduce those frequent sensiless rating changes, the CRI assigns the PDiR by first computing a two-week moving average PD, i.e., 10 working days, and then map it according to the boundary values for different rating categories provided in Table 1.

III. APPLICATION

The PDiR offers a qualitative measure of a firm's credit quality by referencing the historical observed default rates of the S&P rating system. After mapping the CRI one-year PD to the respective PDiR, one is able to have a sense of a firms' credit quality relative to the rating scale adopted by a well-established rating agency. The PDiR offers some operational convenience; for example, one will be able to identify a group of speculative-grade firms based on the standard definition while using the CRI system.

From a time series perspective, one can observe changes in credit quality of a firm over time. By observing the temporal data, the PDiR provides a relative assessment of the firm's credit quality in a conventional way. While the CRI PD offers a high-granularity risk measure for an in-depth analysis of the firm's credit quality, the PDiR presents a convenient and intuitive overview.

Figure 3 below presents the PDiR and the daily CRI 1-year PD of NII Holdings Inc from January 2, 2012 to the business day preceding its default on September 15, 2014. The default occurred about 6 months after its PDiR last dropped to the CCC category.

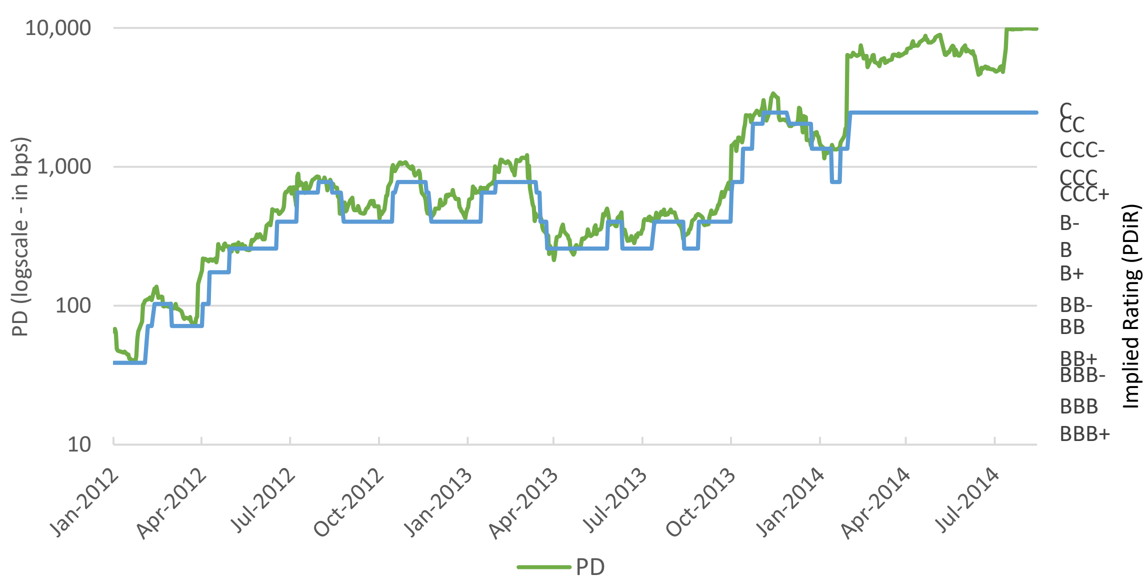

Fig 3. PDiR and 1-year PD of NII Holdings Inc.
Using data as of May 7, 2018. Source: CRI, May 2018.

IV. CONCLUSION

The CRI PDiR complements the CRI 1-year PD by providing a quick and convenient overview on the credit quality of firms in reference to the S&P rating scale. It offers an intuitive understanding of credit risk by benchmarking the PD to historical observed default rates of the S&P rating system.

APPENDIX

Table 2. Conversion table of 1-year PD to PDiR mapping table (based on Moody's observed default rates)

As of May 2018

Rating Category	Observed Moody's Average Default Rate (bps)	Smoothed Moody's Average Default Rate (bps)	CRI PD lower bound (in bps)	CRI PD upper bound (in bps)
Aaa	0.00	0.34	0.00	0.99
Aa1	0.00	1.24	0.99	1.64
Aa2	0.00	1.89	1.64	1.98
Aa3	9.55	2.90	1.98	3.69
A1	8.80	4.44	3.69	6.13
A2	3.20	6.80	6.13	7.68
A3	7.80	10.41	7.68	12.72
Baa1	18.75	15.94	12.72	20.62
Baa2	21.35	24.39	20.62	25.97
Baa3	28.25	37.31	25.97	50.81
Ba1	34.05	57.03	50.81	64.79
Ba2	56.80	87.08	64.79	108.15
Ba3	110.15	132.75	108.15	155.09
B1	125.85	201.88	155.09	304.70
B2	250.30	305.90	304.70	352.27
B3	413.25	460.99	352.27	683.68
Caa1	687.35	689.13	683.68	758.06
Caa2	1344.65	1018.12	758.06	1367.01
Caa3	2188.40	1479.23	1367.01	1705.72
Ca	3187.50	2100.33	1705.72	2332.58
C		3840.93	2332.58	10,000.00

ABOUT THE CREDIT RESEARCH INITIATIVE

The Credit Research Initiative (CRI) was launched by Professor Jin-Chuan Duan in July 2009 at the Risk Management Institute of the National University of Singapore. Aiming at "Transforming Big Data into Smart Data", the CRI covers over 68,000 public firms and produces daily updated Probabilities of Default (1-month to 5-year horizon), Actuarial Spreads (1-year to 5-year contract) and Probability of Default implied Ratings on over 34,000 currently active, exchange-listed firms in 128 economies. The CRI also distributes historical time series of over 34,000 inactive firms due to bankruptcy, corporate consolidation or delisting for other reasons. In addition, the CRI produces and maintains Corporate Vulnerability Indices (CVI), which can be viewed as stress indicators, measuring credit risk in economies, regions and special portfolios.

As a further step, the CRI converts smart data to actionable data to meet the customized demands of its users and offers bespoke credit risk solutions leveraging on its expertise in credit risk analytics. A concrete example is our development of the BuDA (Bottom-up Default Analysis) toolkit in collaboration with the IMF. BuDA is an automated analytic tool based on the CRI PD system, enabling IMF economists to conduct scenarios analyses for the macro-financial linkage.

The CRI publishes Weekly Credit Brief and Quarterly Credit Report, highlighting key credit-related events, offering insights based on the CRI PD of the entities involved, and providing useful statistics on credit risk of economies and sectors.

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